3c. Further Surds (H)

Use your professional judgment in deciding how much foundation tier content to directly teach to higher groups

	Higher
Core	Recap Surds • Understand surd notation • Simplify a surd • simplify and manipulate algebraic expressions with surds by: - collecting like terms - multiplying a single term over a bracket New Learning - Rationalise the denominator where there is one term
Challenge	• simplify and manipulate expressions involving surds by: - taking out common factors - expanding products of two or more binomials New Learning - Rationalise the denominator where there is two terms

What's new?
Rationalising the denominator

36. Inequalities

Use your professional judgment in deciding how much foundation tier content to directly teach to higher groups

	Foundation	Higher
Core	• Show inequalities on number lines; • Write down whole number values that satisfy an inequality; • Use the correct notation to show inclusive and exclusive inequalities; • Solve an inequality such as $-3 < 2x + 1 < 7$ and show the solution set on a number line; • Solve two inequalities in x, find the solution sets and compare them to see which value of x satisfies both; • Construct inequalities to represent a set shown on a number line;	• solve linear inequalities in two variables algebraically;
Challenge		• Represent the solution set for inequalities using set notation, i.e. curly brackets • Solve linear inequalities in two variables graphically; • Show the solution set of several inequalities in two variables on a graph; • Solve quadratic inequalities in one variable, by factorising and sketching the graph to find critical values;

37. Compound Measures

Use your professional judgment in deciding how much foundation tier content to directly teach to higher groups

	Foundation	Higher

Core	• Understand and use compound measures:
	  • density;
	  • pressure;
	  • Speed (mph as well as metric):
	• convert between metric speed measures;
	• read values in km/h and mph from a speedometer;
	
	• Draw distance–time graphs and velocity–time graphs;
	• Work out time intervals for graph scales;
	• Interpret distance–time graphs, and calculate: the speed of individual sections, total distance and total time;
		• convert between density measures
		• convert between pressure measures;

Challenge	• Interpret gradient as the rate of change in distance–time and speed–time graphs • use kinematics formulae from the formulae sheet to calculate speed, acceleration (with variables defined in the question); • change d/t in m/s to a formula in km/h, i.e. d/t × (60 × 60)/1000 – with support;	

Notes/Links

More Real life Graphs content in 41: Real Life Graphs

When you cover kinematic graphs, leave area under a graph and gradient of a curve until 42: Area under & gradient of a curve.

Note that students no longer need to know imperial metric conversions – they are provided in the exam.

39. Bearings & Scale Diagrams

Use your professional judgment in deciding how much foundation tier content to directly teach to higher groups

	Foundation
Core	• Use and interpret maps and scale drawings; • Estimate lengths using a scale diagram; • Make an accurate scale drawing from a diagram; • Use three-figure bearings to specify direction; • Mark on a diagram the position of point *B* given its bearing from point *A*; • Give a bearing between the points on a map or scaled plan; • Given the bearing of a point *A* from point *B*, work out the bearing of *B* from *A*; • Use accurate drawing to solve bearings problems; • Solve locus problems including bearings.
Challenge	Bearings within 35: Right angled Trigonometry

41. Real life graphs

Use your professional judgment in deciding how much foundation tier content to directly teach to higher groups

	Foundation	Higher
Core	• RECAP 15: Coordinates and linear graphs • RECAP 37: compound measures • Read values from straight-line graphs for real-life situations; • Draw straight line graphs for real-life situations, including conversion graphs, fuel bills graphs, fixed charge and cost per unit; • Interpret information presented in a range of linear and non-linear graphs; • Interpret graphs with negative values on axes;	• RECAP 15: Coordinates and linear graphs • RECAP 37: compound measures • Use graphs to calculate various measures (of individual sections), including: unit price (gradient), average speed, distance, time, acceleration; including using enclosed areas by counting squares or using areas of trapezia, rectangles and triangles;
Challenge	Interpret gradient as the rate of change in graphs of containers filling and emptying, and unit price graphs.	• Interpret and analyse information presented in a range of linear graphs: • use gradients to interpret how one variable changes in relation to another;

Notes/Links

Higher includes exponential graphs – should be covered in 38: Growth and decay.

Distance-time and speed-time graphs should have already been taught (under 37: compound measures).

This blog post provides more details about what you need to teach

When you cover kinematic graphs, leave estimating area under a graph and estimating gradient of a curve until 42: Area under & gradient of a curve

42. Area under & gradient of a curve

This is a Higher Tier topic – Foundation Tier teachers should recap topics as guided below

	Foundation	Higher
Core	RECAP 12: Area and perimeter 37: Compound measures 41: Real life graphs	RECAP 12: Area and perimeter 37: Compound measures 41: Real life graphs • Estimate area under a quadratic or other graph by dividing it into trapezia; • Estimate the gradient of a quadratic or non-linear graph at a given point by sketching the tangent and finding its gradient; • for a non-linear distance–time (or velocity-time) graph, estimate the speed (acceleration) at one point in time, from the tangent, and the average speed (acceleration) over several seconds by finding the gradient of the chord;
Challenge		• Interpret the gradient of or area under a linear or non-linear graph in a variety of real-life contexts: financial, containers filling and emptying, unit prices changing

43. Trigonometry

Use your professional judgment in deciding how much foundation tier content to directly teach to higher groups

	Foundation	Higher
Core	• Recap 35: Right-angled trigonometry	• Understand, recall and use trigonometric relationships and Pythagoras' Theorem in right-angled triangles, and use these to solve problems in 3D configurations, eg: find the angle between a line and a plane.
Challenge		• Know and apply Area $=\frac{1}{2}ab\sin C$ to calculate the area, sides or angles of any triangle. • Know the sine and cosine rules, and use to solve 2D problems (including involving bearings). • Use the sine and cosine rules to solve 3D problems. • Understand the language of planes, and recognise the diagonals of a cuboid.

44. Congruence and similarity

Use your professional judgment in deciding how much foundation tier content to directly teach to higher groups

	Foundation	Higher
Core	• Understand congruence and visually identify congruent shapes • Understand, from experience of constructing them, that triangles satisfying SSS, SAS, ASA and RHS are unique, but SSA triangles are not; • Use the basic congruence criteria for triangles • Solve angle problems involving congruence; • Identify shapes which are similar • Understand similarity of triangles and of other plane shapes, use this to make geometric inferences, and solve angle problems using similarity;	• Prove that two shapes are similar by showing that all corresponding angles are equal in size and/or lengths of sides are in the same ratio/one is an enlargement of the other, giving the scale factor; • Formal proofs of congruence • Solve angle problems by first proving congruence; • Understand the effect of enlargement on angles, perimeter, area and volume of shapes and solids;
Challenge	• Identify the scale factor of an enlargement of a shape as the ratio of the lengths of two corresponding sides; • Understand the effect of enlargement on perimeter of shapes; • Solve problems to find missing lengths in similar shapes; • Know that scale diagrams, including bearings and maps are 'similar' to the real-life examples.	• Use formal geometric proof for the similarity of two given triangles; • Write the lengths, areas and volumes of two shapes as ratios in their simplest form; • Find missing lengths, areas and volumes in similar 3D solids; • Know the relationships between linear, area and volume scale factors of mathematically similar shapes and solids; • Solve problems involving frustums of cones where you have to find missing lengths first using similar triangles.

45. Circle Theorems

This is a Higher Tier topic – Foundation Tier teachers should recap topics as guided below

	Foundation	Higher

Core	RECAP
	13: Circumference and area
	24: Angles
		• Recall the definition of a circle and identify (name) and draw parts of a circle, including sector, tangent, chord, segment;
		• Use the facts that:
		  • the angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the circumference;
		  • the angle in a semicircle is a right angle;
		  • the perpendicular from the centre of a circle to a chord bisects the chord;
		  • angles in the same segment are equal;
		  • alternate segment theorem;
		  • opposite angles of a cyclic quadrilateral sum to 180°;
		  • the tangent at any point on a circle is perpendicular to the radius at that point;
		  • the fact that tangents from an external point are equal in length.
		• Find and give reasons for missing angles on diagrams using:
		  • circle theorems;
		  • isosceles triangles (radius properties) in circles;

Challenge		• Proofs of circle theorems

46. Algebraic Fractions

This is a Higher Tier topic – Foundation Tier teachers should recap topics as guided below

	Foundation	Higher
Core	RECAP 5: Algebra 6: Equations and formulae 17: Fractions	RECAP 5: Algebra 6: Equations and formulae 17: Fractions
Challenge		• Multiply and divide algebraic fractions; • Solve quadratic equations arising from algebraic fraction equations; • Change the subject of a formula such as $\frac{2}{x+m} = \frac{3}{2x-m}$, where all variables are in the denominators;

47. Proof

Use your professional judgment in deciding how much foundation tier content to directly teach to higher groups

	Foundation	Higher
Core	6. Equations and formulae 24. Angles 45. Congruence and similarity	• *Simple proofs and use of ≡ in "show that" style questions;*
Challenge	• Understand a proof that the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices; Use geometrical language appropriately, give reasons for angle calculations and show step-by-step deduction when solving problems • Find out/prove whether two variables are in direct proportion by plotting the graph and using it as a model to read off other values. • Visual proof of the difference of two squares • Answer 'show that' questions using consecutive integers (n, $n+1$), squares a^2, b^2, even numbers $2n$, and odd numbers $2n+1$; • Arguments may use knowledge of odd and even, for example odd x even = even • Students should understand that, for example if n is an integer then $2n$ is even and $2n+1$ is odd. Work out the values of a and b in the identity $2(ax-5)+3(5x+b) \equiv 21x+2$ Show that $3(a-4)+2(2a+5)+9$ and $7(a+1)$ are equivalent. w is an even number. Explain why $(w-1)(w+1)$ will always be odd. Sam says that when $m > 1$, m^3+2 is never a multiple of 3 Give an example to show that she is wrong.	• Construct rigorous proofs to validate a given result Examples 1 Alice says that the sum of three consecutive numbers will always be even. Explain why she is wrong. 2 Prove that the product of three consecutive positive integers must always be a multiple of 6 3 n is a positive integer. Explain why $n(n+1)$ must be an even number. 4 Prove that the square of any odd number is always one more than a multiple of 8 5 (a) Show that $\frac{2x+5}{3}+\frac{3x-1}{2}$ simplifies to $\frac{13x+7}{6}$ (b) Hence solve $\frac{2x+5}{3}+\frac{3x-1}{2}=8$ 6 Prove that $(2x+3)^2-3x(x+4)$ is always positive.

Notes/Links
Geometric proofs should have been covered elsewhere, but it might be worth returning to them here eg similarity proofs, congruency proofs, circle theorems, sum of angles in a triangle etc

48. Vectors

Use your professional judgment in deciding how much foundation tier content to directly teach to higher groups

	Foundation	Higher
Core	• Understand and use column notation in relation to vectors; • Be able to represent information graphically given column vectors; • Identify two column vectors which are parallel;	• Understand and interpret vectors as displacement in the plane with an associated direction. • Understand that 2**a** is parallel to **a** and twice its length, and that **a** is parallel to –**a** in the opposite direction. • Represent vectors, combinations of vectors and scalar multiples in the plane pictorially. • Calculate the sum of two vectors, the difference of two vectors and a scalar multiple of a vector using column vectors (including algebraic terms). • Find the length of a vector using Pythagoras' Theorem. • Calculate the resultant of two vectors. • Solve geometric problems in 2D where vectors are divided in a given ratio.
Challenge	• Calculate using column vectors, and represent graphically, the sum of two vectors, the difference of two vectors and a scalar multiple of a vector.	• Produce geometrical proofs to prove points are collinear and vectors/lines are parallel.

49. Equation of circle & tangent

This is a Higher Tier topic – Foundation Tier teachers should recap topics as guided below

	Foundation	Higher
Core	RECAP 15: Coordinate and linear graphs 39: Graph and graph transformations	RECAP 15: Coordinate and linear graphs 39: Graph and graph transformations
		• Select and apply construction techniques and understanding of loci to draw graphs based on circles and perpendiculars of lines; • Recognise and construct the graph of a circle using $x^2 + y^2 = r^2$ for radius r **centred at the origin** of coordinates

Challenge		• Find the equation of a tangent to a circle at a given point, by:
		  • finding the gradient of the radius that meets the circle at that point (circles all centre the origin);
		  • finding the gradient of the tangent perpendicular to it;
		  • using the given point;