

Linear inequalities

A LEVEL LINKS

Scheme of work: 1d. Inequalities – linear and quadratic (including graphical solutions)

Key points

- Solving linear inequalities uses similar methods to those for solving linear equations.
- When you multiply or divide an inequality by a negative number you need to reverse the inequality sign, e.g. $<$ becomes $>$.

Examples

Example 1 Solve $-8 \leq 4x < 16$

$\begin{aligned} -8 &\leq 4x < 16 \\ -2 &\leq x < 4 \end{aligned}$	Divide all three terms by 4.
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Example 2 Solve $4 \leq 5x < 10$

$\begin{aligned} 4 &\leq 5x < 10 \\ \frac{4}{5} &\leq x < 2 \end{aligned}$	Divide all three terms by 5.
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Example 3 Solve $2x - 5 < 7$

$\begin{aligned} 2x - 5 &< 7 \\ 2x &< 12 \\ x &< 6 \end{aligned}$	<ol style="list-style-type: none"> Add 5 to both sides. Divide both sides by 2.
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Example 4 Solve $2 - 5x \geq -8$

$\begin{aligned} 2 - 5x &\geq -8 \\ -5x &\geq -10 \\ x &\leq 2 \end{aligned}$	<ol style="list-style-type: none"> Subtract 2 from both sides. Divide both sides by -5. Remember to reverse the inequality when dividing by a negative number.
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Example 5 Solve $4(x - 2) > 3(9 - x)$

$\begin{aligned} 4(x - 2) &> 3(9 - x) \\ 4x - 8 &> 27 - 3x \\ 7x - 8 &> 27 \\ 7x &> 35 \\ x &> 5 \end{aligned}$	<ol style="list-style-type: none"> Expand the brackets. Add $3x$ to both sides. Add 8 to both sides. Divide both sides by 7.
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Practice

1 Solve these inequalities.

a $4x > 16$

b $5x - 7 \leq 3$

c $1 \geq 3x + 4$

d $5 - 2x < 12$

e $\frac{x}{2} \geq 5$

f $8 < 3 - \frac{x}{3}$

2 Solve these inequalities.

a $\frac{x}{5} < -4$

b $10 \geq 2x + 3$

c $7 - 3x > -5$

3 Solve

a $2 - 4x \geq 18$

b $3 \leq 7x + 10 < 45$

c $6 - 2x \geq 4$

d $4x + 17 < 2 - x$

e $4 - 5x < -3x$

f $-4x \geq 24$

4 Solve these inequalities.

a $3t + 1 < t + 6$

b $2(3n - 1) \geq n + 5$

5 Solve.

a $3(2 - x) > 2(4 - x) + 4$

b $5(4 - x) > 3(5 - x) + 2$

Extend

6 Find the set of values of x for which $2x + 1 > 11$ and $4x - 2 > 16 - 2x$.

Answers

1 a $x > 4$

d $x > -\frac{7}{2}$

b $x \leq 2$

e $x \geq 10$

c $x \leq -1$

f $x < -15$

2 a $x < -20$

b $x \leq 3.5$

c $x < 4$

3 a $x \leq -4$

d $x < -3$

b $-1 \leq x < 5$

e $x > 2$

c $x \leq 1$

f $x \leq -6$

4 a $t < \frac{5}{2}$

b $n \geq \frac{7}{5}$

5 a $x < -6$

b $x < \frac{3}{2}$

6 $x > 5$ (which also satisfies $x > 3$)