

Rearranging equations

A LEVEL LINKS

Scheme of work: 6a. Definition, differentiating polynomials, second derivatives

Textbook: Pure Year 1, 12.1 Gradients of curves

Key points

- To change the subject of a formula, get the terms containing the subject on one side and everything else on the other side.
- You may need to factorise the terms containing the new subject.

Examples

Example 1 Make t the subject of the formula $v = u + at$.

$v = u + at$ $v - u = at$ $t = \frac{v - u}{a}$	<ol style="list-style-type: none"> 1 Get the terms containing t on one side and everything else on the other side. 2 Divide throughout by a.
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Example 2 Make t the subject of the formula $r = 2t - \pi t$.

$r = 2t - \pi t$ $r = t(2 - \pi)$ $t = \frac{r}{2 - \pi}$	<ol style="list-style-type: none"> 1 All the terms containing t are already on one side and everything else is on the other side. 2 Factorise as t is a common factor. 3 Divide throughout by $2 - \pi$.
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Example 3 Make t the subject of the formula $\frac{t + r}{5} = \frac{3t}{2}$.

$\frac{t + r}{5} = \frac{3t}{2}$ $2t + 2r = 15t$ $2r = 13t$ $t = \frac{2r}{13}$	<ol style="list-style-type: none"> 1 Remove the fractions first by multiplying throughout by 10. 2 Get the terms containing t on one side and everything else on the other side and simplify. 3 Divide throughout by 13.
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Example 4 Make t the subject of the formula $r = \frac{3t+5}{t-1}$.

$r = \frac{3t+5}{t-1}$ $r(t-1) = 3t+5$ $rt - r = 3t+5$ $rt - 3t = 5 + r$ $t(r-3) = 5 + r$ $t = \frac{5+r}{r-3}$	<ol style="list-style-type: none"> 1 Remove the fraction first by multiplying throughout by $t-1$. 2 Expand the brackets. 3 Get the terms containing t on one side and everything else on the other side. 4 Factorise the LHS as t is a common factor. 5 Divide throughout by $r-3$.
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Practice

Change the subject of each formula to the letter given in the brackets.

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| 1 $C = \pi d$ [d] | 2 $P = 2l + 2w$ [w] | 3 $D = \frac{S}{T}$ [T] |
| 4 $p = \frac{q-r}{t}$ [t] | 5 $u = at - \frac{1}{2}t$ [t] | 6 $V = ax + 4x$ [x] |
| 7 $\frac{y-7x}{2} = \frac{7-2y}{3}$ [y] | 8 $x = \frac{2a-1}{3-a}$ [a] | 9 $x = \frac{b-c}{d}$ [d] |
| 10 $h = \frac{7g-9}{2+g}$ [g] | 11 $e(9+x) = 2e+1$ [e] | 12 $y = \frac{2x+3}{4-x}$ [x] |

13 Make r the subject of the following formulae.

a $A = \pi r^2$	b $V = \frac{4}{3}\pi r^3$	c $P = \pi r + 2r$	d $V = \frac{2}{3}\pi r^2 h$
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14 Make x the subject of the following formulae.

a $\frac{xy}{z} = \frac{ab}{cd}$	b $\frac{4\pi cx}{d} = \frac{3z}{py^2}$
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15 Make $\sin B$ the subject of the formula $\frac{a}{\sin A} = \frac{b}{\sin B}$

16 Make $\cos B$ the subject of the formula $b^2 = a^2 + c^2 - 2ac \cos B$.

Extend

17 Make x the subject of the following equations.

a $\frac{p}{q}(sx+t) = x-1$	b $\frac{p}{q}(ax+2y) = \frac{3p}{q^2}(x-y)$
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Answers

$$1 \quad d = \frac{C}{\pi}$$

$$2 \quad w = \frac{P-2l}{2}$$

$$3 \quad T = \frac{S}{D}$$

$$4 \quad t = \frac{q-r}{p}$$

$$5 \quad t = \frac{2u}{2a-1}$$

$$6 \quad x = \frac{V}{a+4}$$

$$7 \quad y = 2 + 3x$$

$$8 \quad a = \frac{3x+1}{x+2}$$

$$9 \quad d = \frac{b-c}{x}$$

$$10 \quad g = \frac{2h+9}{7-h}$$

$$11 \quad e = \frac{1}{x+7}$$

$$12 \quad x = \frac{4y-3}{2+y}$$

$$13 \quad \mathbf{a} \quad r = \sqrt{\frac{A}{\pi}}$$

$$\mathbf{b} \quad r = \sqrt[3]{\frac{3V}{4\pi}}$$

$$\mathbf{c} \quad r = \frac{P}{\pi+2}$$

$$\mathbf{d} \quad r = \sqrt{\frac{3V}{2\pi h}}$$

$$14 \quad \mathbf{a} \quad x = \frac{abz}{cdy}$$

$$\mathbf{b} \quad x = \frac{3dz}{4\pi cpy^2}$$

$$15 \quad \sin B = \frac{b \sin A}{a}$$

$$16 \quad \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$17 \quad \mathbf{a} \quad x = \frac{q+pt}{q-ps}$$

$$\mathbf{b} \quad x = \frac{3py+2pqy}{3p-apq} = \frac{y(3+2q)}{3-aq}$$