Solving linear simultaneous equations using the elimination method

A LEVEL LINKS
Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

- Two equations are simultaneous when they are both true at the same time.
- Solving simultaneous linear equations in two unknowns involves finding the value of each unknown which works for both equations.
- Make sure that the coefficient of one of the unknowns is the same in both equations.
- Eliminate this equal unknown by either subtracting or adding the two equations.

Examples

Example 1 Solve the simultaneous equations $3x + y = 5$ and $x + y = 1$

$\begin{aligned} 3x + y &= 5 \\ - \quad x + y &= 1 \\ \hline 2x \quad\;\; &= 4 \end{aligned}$ So $x = 2$	**1** Subtract the second equation from the first equation to eliminate the y term.
Using $x + y = 1$ $\qquad 2 + y = 1$ So $y = -1$	**2** To find the value of y, substitute $x = 2$ into one of the original equations.
Check: $\quad$ equation 1: $3 \times 2 + (-1) = 5$ YES $\quad$ equation 2: $2 + (-1) = 1$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Example 2 Solve $x + 2y = 13$ and $5x - 2y = 5$ simultaneously.

$\begin{aligned} x + 2y &= 13 \\ + \;\; 5x - 2y &= \;\; 5 \\ \hline 6x \qquad &= 18 \end{aligned}$ So $x = 3$	**1** Add the two equations together to eliminate the y term.
Using $x + 2y = 13$ $\qquad 3 + 2y = 13$ So $y = 5$	**2** To find the value of y, substitute $x = 3$ into one of the original equations.
Check: $\quad$ equation 1: $3 + 2 \times 5 = 13$ YES $\quad$ equation 2: $5 \times 3 - 2 \times 5 = 5$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Example 3   Solve $2x + 3y = 2$ and $5x + 4y = 12$ simultaneously.

$(2x + 3y = 2) \times 4 \rightarrow \quad 8x + 12y = 8$ $(5x + 4y = 12) \times 3 \rightarrow \quad \underline{15x + 12y = 36}$ $\qquad\qquad\qquad\qquad 7x \qquad = 28$ So $x = 4$	**1** Multiply the first equation by 4 and the second equation by 3 to make the coefficient of y the same for both equations. Then subtract the first equation from the second equation to eliminate the y term.
Using $\ 2x\ +\ 3y\ = 2$ $\qquad\ 2 \times 4 + 3y = 2$ So $y = -2$	**2** To find the value of y, substitute $x = 4$ into one of the original equations.
Check: $\quad$ equation 1: $2 \times 4 + 3 \times (-2) = 2 \quad$ YES $\quad$ equation 2: $5 \times 4 + 4 \times (-2) = 12 \quad$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Practice

Solve these simultaneous equations.

1 $\quad 4x + y = 8$
$\qquad x + y = 5$

2 $\quad 3x + y = 7$
$\qquad 3x + 2y = 5$

3 $\quad 4x + y = 3$
$\qquad 3x - y = 11$

4 $\quad 3x + 4y = 7$
$\qquad x - 4y = 5$

5 $\quad 2x + y = 11$
$\qquad x - 3y = 9$

6 $\quad 2x + 3y = 11$
$\qquad 3x + 2y = 4$

Solving linear simultaneous equations using the substitution method

> **A LEVEL LINKS**
> **Scheme of work:** 1c. Equations – quadratic/linear simultaneous
> **Textbook:** Pure Year 1, 3.1 Linear simultaneous equations

Key points

- The subsitution method is the method most commonly used for A level. This is because it is the method used to solve linear and quadratic simultaneous equations.

Examples

Example 4   Solve the simultaneous equations $y = 2x + 1$ and $5x + 3y = 14$

$5x + 3(2x + 1) = 14$	**1** Substitute $2x + 1$ for y into the second equation.
$5x + 6x + 3 = 14$	**2** Expand the brackets and simplify.
$11x + 3 = 14$	
$11x = 11$	**3** Work out the value of x.
So $x = 1$	
Using $y = 2x + 1$	**4** To find the value of y, substitute
$\qquad y = 2 \times 1 + 1$	$x = 1$ into one of the original
So $y = 3$	equations.
Check:	**5** Substitute the values of x and y into
equation 1: $3 = 2 \times 1 + 1$ YES	both equations to check your
equation 2: $5 \times 1 + 3 \times 3 = 14$ YES	answers.

Example 5   Solve $2x - y = 16$ and $4x + 3y = -3$ simultaneously.

$y = 2x - 16$	**1** Rearrange the first equation.
$4x + 3(2x - 16) = -3$	**2** Substitute $2x - 16$ for y into the second equation.
$4x + 6x - 48 = -3$	**3** Expand the brackets and simplify.
$10x - 48 = -3$	
$10x = 45$	**4** Work out the value of x.
So $x = 4\frac{1}{2}$	
Using $y = 2x - 16$	
$\qquad y = 2 \times 4\frac{1}{2} - 16$	**5** To find the value of y, substitute $x = 4\frac{1}{2}$ into one of the original equations.
So $y = -7$	
Check:	**6** Substitute the values of x and y into both equations to check your answers.
equation 1: $2 \times 4\frac{1}{2} - (-7) = 16$ YES	
equation 2: $4 \times 4\frac{1}{2} + 3 \times (-7) = -3$ YES	

Practice

Solve these simultaneous equations.

7 $y = x - 4$
  $2x + 5y = 43$

8 $y = 2x - 3$
  $5x - 3y = 11$

9 $2y = 4x + 5$
  $9x + 5y = 22$

10 $2x = y - 2$
  $8x - 5y = -11$

11 $3x + 4y = 8$
  $2x - y = -13$

12 $3y = 4x - 7$
  $2y = 3x - 4$

13 $3x = y - 1$
  $2y - 2x = 3$

14 $3x + 2y + 1 = 0$
  $4y = 8 - x$

Extend

15 Solve the simultaneous equations $3x + 5y - 20 = 0$ and $2(x + y) = \dfrac{3(y - x)}{4}$.

Answers

1 $x = 1, y = 4$

2 $x = 3, y = -2$

3 $x = 2, y = -5$

4 $x = 3, y = -\frac{1}{2}$

5 $x = 6, y = -1$

6 $x = -2, y = 5$

7 $x = 9, y = 5$

8 $x = -2, y = -7$

9 $x = \frac{1}{2}, y = 3\frac{1}{2}$

10 $x = \frac{1}{2}, y = 3$

11 $x = -4, y = 5$

12 $x = -2, y = -5$

13 $x = \frac{1}{4}, y = 1\frac{3}{4}$

14 $x = -2, y = 2\frac{1}{2}$

15 $x = -2\frac{1}{2}, y = 5\frac{1}{2}$