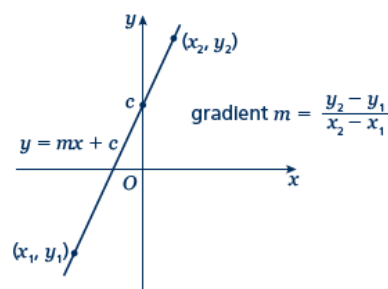# Straight line graphs

A LEVEL LINKS
Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

- A straight line has the equation $y = mx + c$, where m is the gradient and c is the y-intercept (where $x = 0$).
- The equation of a straight line can be written in the form $ax + by + c = 0$, where a, b and c are integers.
- When given the coordinates (x_1, y_1) and (x_2, y_2) of two points on a line the gradient is calculated using the formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Examples

Example 1   A straight line has gradient $-\dfrac{1}{2}$ and y-intercept 3.

Write the equation of the line in the form $ax + by + c = 0$.

$m = -\dfrac{1}{2}$ and $c = 3$ So $y = -\dfrac{1}{2}x + 3$ $\dfrac{1}{2}x + y - 3 = 0$ $x + 2y - 6 = 0$	**1** A straight line has equation $y = mx + c$. Substitute the gradient and y-intercept given in the question into this equation. **2** Rearrange the equation so all the terms are on one side and 0 is on the other side. **3** Multiply both sides by 2 to eliminate the denominator.

Example 2   Find the gradient and the y-intercept of the line with the equation $3y - 2x + 4 = 0$.

$3y - 2x + 4 = 0$ $3y = 2x - 4$ $y = \dfrac{2}{3}x - \dfrac{4}{3}$ Gradient $= m = \dfrac{2}{3}$ y-intercept $= c = -\dfrac{4}{3}$	**1** Make y the subject of the equation. **2** Divide all the terms by three to get the equation in the form $y = \dots$ **3** In the form $y = mx + c$, the gradient is m and the y-intercept is c.

Example 3 Find the equation of the line which passes through the point (5, 13) and has gradient 3.

$m = 3$ $y = 3x + c$ $13 = 3 \times 5 + c$ $13 = 15 + c$ $c = -2$ $y = 3x - 2$	**1** Substitute the gradient given in the question into the equation of a straight line $y = mx + c$. **2** Substitute the coordinates $x = 5$ and $y = 13$ into the equation. **3** Simplify and solve the equation. **4** Substitute $c = -2$ into the equation $y = 3x + c$

Example 4 Find the equation of the line passing through the points with coordinates (2, 4) and (8, 7).

$x_1 = 2$, $x_2 = 8$, $y_1 = 4$ and $y_2 = 7$ $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{7 - 4}{8 - 2} = \dfrac{3}{6} = \dfrac{1}{2}$ $y = \dfrac{1}{2}x + c$ $4 = \dfrac{1}{2} \times 2 + c$ $c = 3$ $y = \dfrac{1}{2}x + 3$	**1** Substitute the coordinates into the equation $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line. **2** Substitute the gradient into the equation of a straight line $y = mx + c$. **3** Substitute the coordinates of either point into the equation. **4** Simplify and solve the equation. **5** Substitute $c = 3$ into the equation $y = \dfrac{1}{2}x + c$

Practice

1 Find the gradient and the *y*-intercept of the following equations.

    a $y = 3x + 5$ **b** $y = -\dfrac{1}{2}x - 7$

    c $2y = 4x - 3$ **d** $x + y = 5$

    e $2x - 3y - 7 = 0$ **f** $5x + y - 4 = 0$

> **Hint**
> Rearrange the equations to the form $y = mx + c$

2 Copy and complete the table, giving the equation of the line in the form $y = mx + c$.

Gradient	*y*-intercept	Equation of the line
5	0	
–3	2	
4	–7	

3   Find, in the form $ax + by + c = 0$ where a, b and c are integers, an equation for each of the lines with the following gradients and y-intercepts.

    a   gradient $-\dfrac{1}{2}$, y-intercept -7 **b**   gradient 2, y-intercept 0

    c   gradient $\dfrac{2}{3}$, y-intercept 4   **d**   gradient -1.2, y-intercept -2

4   Write an equation for the line which passes though the point (2, 5) and has gradient 4.

5   Write an equation for the line which passes through the point (6, 3) and has gradient $-\dfrac{2}{3}$

6   Write an equation for the line passing through each of the following pairs of points.
    a   (4, 5), (10, 17)   **b**   (0, 6), (–4, 8)
    c   (–1, –7), (5, 23)   **d**   (3, 10), (4, 7)

Extend

7   The equation of a line is $2y + 3x - 6 = 0$.
Write as much information as possible about this line.

Answers

1 **a** $m = 3, c = 5$ **b** $m = -\dfrac{1}{2}, c = -7$

     c $m = 2, c = -\dfrac{3}{2}$ **d** $m = -1, c = 5$

     e $m = \dfrac{2}{3}, c = -\dfrac{7}{3} \text{ or } -2\dfrac{1}{3}$ **f** $m = -5, c = 4$

2

Gradient	*y*-intercept	Equation of the line
5	0	$y = 5x$
−3	2	$y = -3x + 2$
4	−7	$y = 4x - 7$

3 **a** $x + 2y + 14 = 0$ **b** $2x - y = 0$

     c $2x - 3y + 12 = 0$ **d** $6x + 5y + 10 = 0$

4 $y = 4x - 3$

5 $y = -\dfrac{2}{3}x + 7$

6 **a** $y = 2x - 3$ **b** $y = -\dfrac{1}{2}x + 6$

     c $y = 5x - 2$ **d** $y = -3x + 19$

7 $y = -\dfrac{3}{2}x + 3$, the gradient is $-\dfrac{3}{2}$ and the *y*-intercept is 3.

The line intercepts the axes at (0, 3) and (2, 0).

Students may sketch the line or give coordinates that lie on the line such as $\left(1, \dfrac{3}{2}\right)$ or $\left(4, -3\right)$.