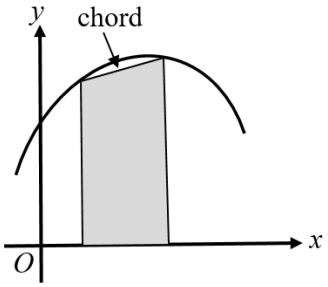
Area under a graph

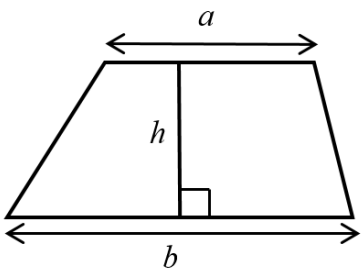
A LEVEL LINKS
Scheme of work: 7b. Definite integrals and areas under curves

Key points

- To estimate the area under a curve, draw a chord between the two points you are finding the area between and straight lines down to the horizontal axis to create a trapezium. The area of the trapezium is an approximation for the area under a curve.

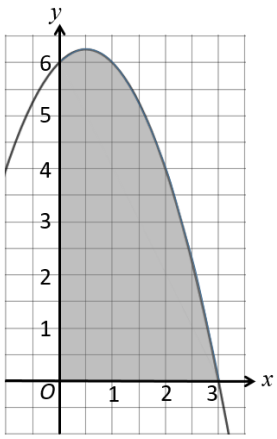


- The area of a trapezium = $\frac{1}{2}h(a+b)$



Examples

Example 1 Estimate the area of the region between the curve $y = (3 - x)(2 + x)$ and the x-axis from $x = 0$ to $x = 3$. Use three strips of width 1 unit.



x	0	1	2	3
$y = (3 - x)(2 + x)$	6	6	4	0

Trapezium 1:
$a_1 = 6 - 0 = 6,\ b_1 = 6 - 0 = 6$
Trapezium 2:
$a_2 = 6 - 0 = 6,\ b_2 = 4 - 0 = 4$
Trapezium 3:
$a_3 = 4 - 0 = 4,\ a_3 = 0 - 0 = 0$

1 Use a table to record the value of y on the curve for each value of x.

2 Work out the dimensions of each trapezium. The distances between the y-values on the curve and the x-axis give the values for a.

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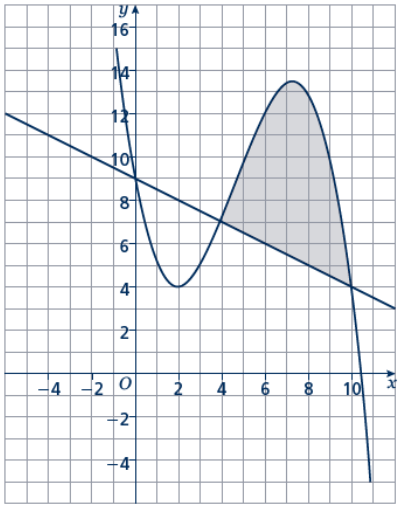
$\frac{1}{2}h(a_1+b_1)=\frac{1}{2}\times1(6+6)=6$ $\frac{1}{2}h(a_2+b_2)=\frac{1}{2}\times1(6+4)=5$ $\frac{1}{2}h(a_3+b_3)=\frac{1}{2}\times1(4+0)=2$ Area = 6 + 5 + 2 = 13 units^2	**3** Work out the area of each trapezium. h = 1 since the width of each trapezium is 1 unit. **4** Work out the total area. Remember to give units with your answer.

Example 2 Estimate the shaded area.
Use three strips of width 2 units.



x	4	6	8	10
y	7	12	13	4

x	4	6	8	10
y	7	6	5	4

Trapezium 1:
$a_1=7-7=0,\ b_1=12-6=6$
Trapezium 2:
$a_2=12-6=6,\ b_2=13-5=8$
Trapezium 3:
$a_3=13-5=8,\ a_3=4-4=0$

$\frac{1}{2}h(a_1+b_1)=\frac{1}{2}\times2(0+6)=6$
$\frac{1}{2}h(a_2+b_2)=\frac{1}{2}\times2(6+8)=14$
$\frac{1}{2}h(a_3+b_3)=\frac{1}{2}\times2(8+0)=8$

Area = 6 + 14 + 8 = 28 units^2

1 Use a table to record y on the curve for each value of x.

2 Use a table to record y on the straight line for each value of x.

3 Work out the dimensions of each trapezium. The distances between the y-values on the curve and the y-values on the straight line give the values for a.

4 Work out the area of each trapezium. h = 2 since the width of each trapezium is 2 units.

5 Work out the total area. Remember to give units with your answer.

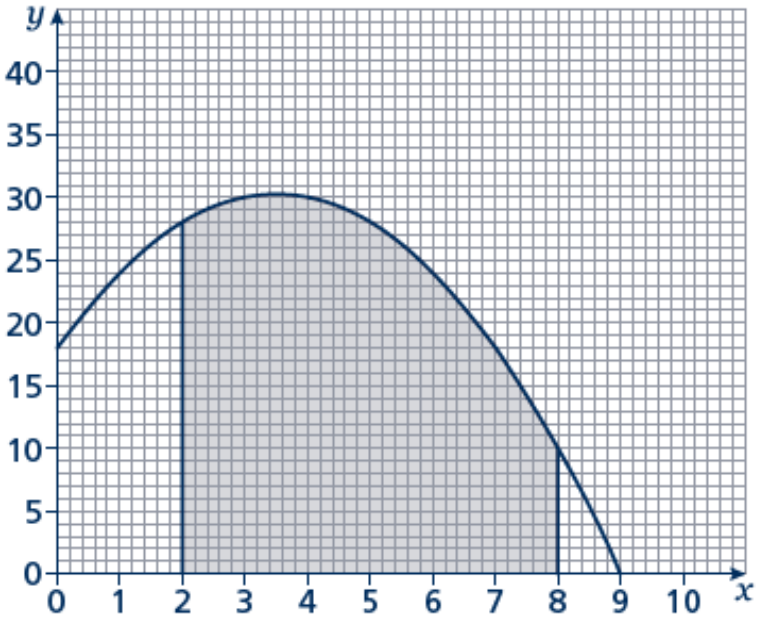
Practice

Hint:

For a full answer, remember to include 'units^2'.

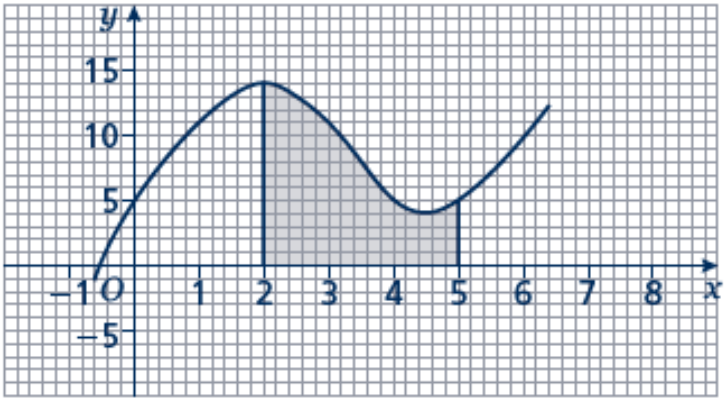
1 Estimate the area of the region between the curve $y = (5 - x)(x + 2)$ and the x-axis from $x = 1$ to $x = 5$.
Use four strips of width 1 unit.

2 Estimate the shaded area shown on the axes.
Use six strips of width 1 unit.



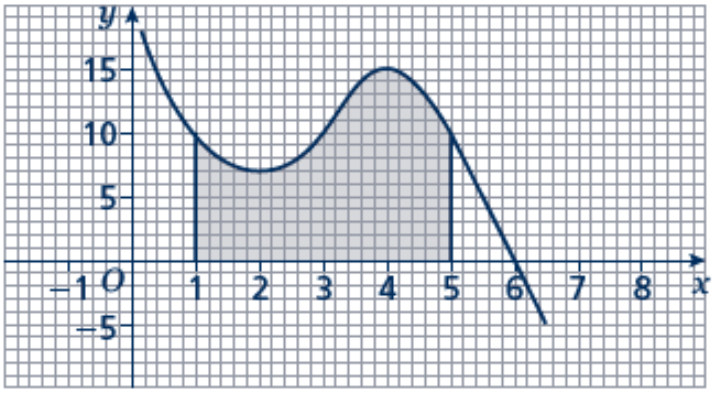
3 Estimate the area of the region between the curve $y = x^2 - 8x + 18$ and the x-axis from $x = 2$ to $x = 6$.
Use four strips of width 1 unit.

4 Estimate the shaded area.
Use six strips of width $\frac{1}{2}$ unit.



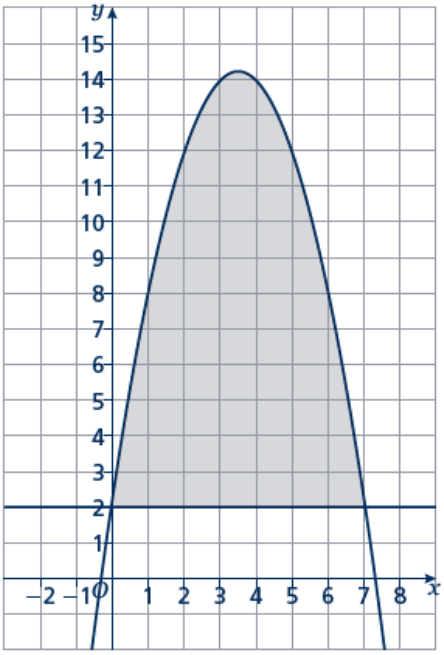
5 Estimate the area of the region between the curve $y = -x^2 - 4x + 5$ and the x-axis from $x = -5$ to $x = 1$.
Use six strips of width 1 unit.

6 Estimate the shaded area.
Use four strips of equal width.



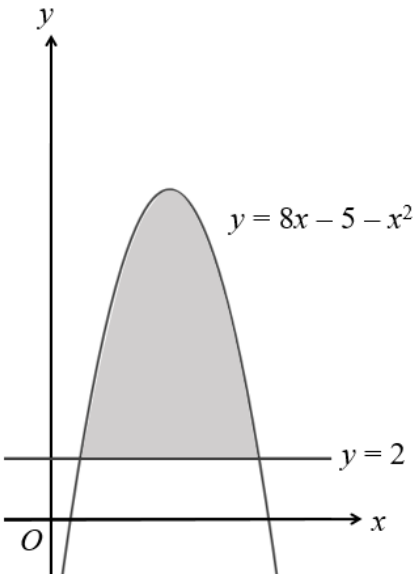
7 Estimate the area of the region between the curve $y = -x^2 + 2x + 15$ and the x-axis from $x = 2$ to $x = 5$.
Use six strips of equal width.

8 Estimate the shaded area.
Use seven strips of equal width.

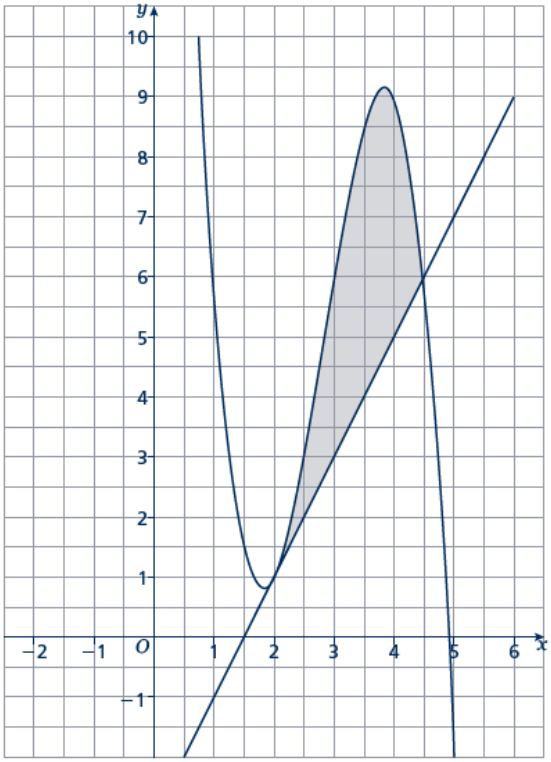


Extend

9 The curve $y = 8x - 5 - x^2$ and the line $y = 2$ are shown in the sketch.
Estimate the shaded area using six strips of equal width.



10 Estimate the shaded area using five strips of equal width.



Answers

1 34 units^2

2 149 units^2

3 14 units^2

4 $25\frac{1}{4}$ units^2

5 35 units^2

6 42 units^2

7 $26\frac{7}{8}$ units^2

8 56 units^2

9 35 units^2

10 $6\frac{1}{4}$ units^2