Translating graphs

Key points

- The transformation $y = f(x) \pm a$ is a translation of $y = f(x)$ parallel to the y-axis; it is a vertical translation.
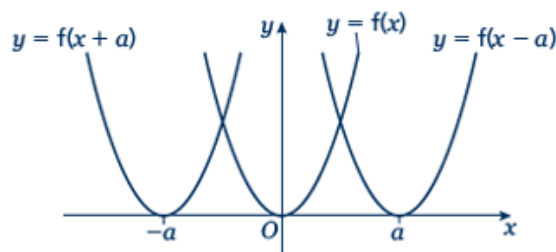
  As shown on the graph,
  - $y = f(x) + a$ translates $y = f(x)$ up
  - $y = f(x) - a$ translates $y = f(x)$ down.
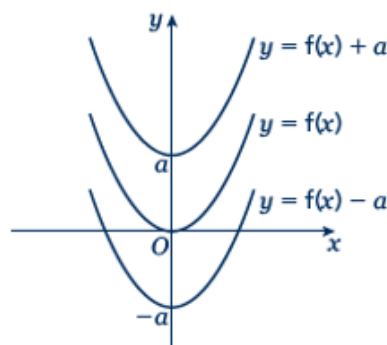
- The transformation $y = f(x \pm a)$ is a translation of $y = f(x)$ parallel to the x-axis; it is a horizontal translation.
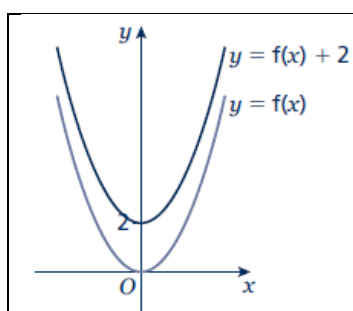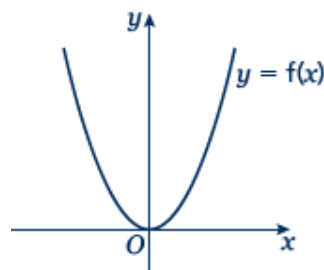
  As shown on the graph,
  - $y = f(x + a)$ translates $y = f(x)$ to the left
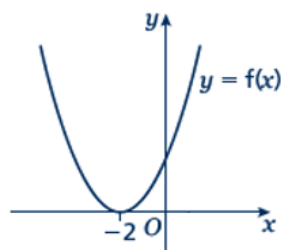 - $y = f(x - a)$ translates $y = f(x)$ to the right.

Examples

Example 1 The graph shows the function $y = f(x)$.
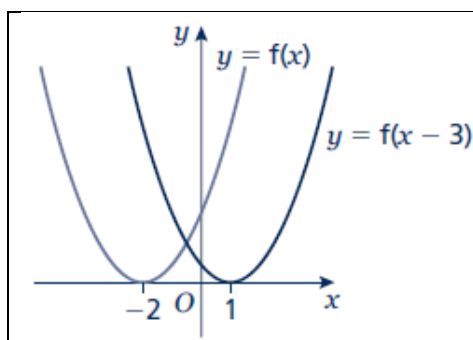
Sketch the graph of $y = f(x) + 2$.

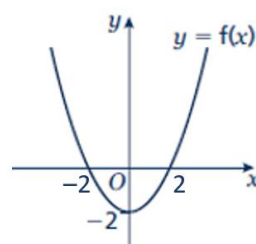|	
(graph showing $y = f(x) + 2$ and $y = f(x)$)	For the function $y = f(x) + 2$ translate the function $y = f(x)$ 2 units up.

Example 2 The graph shows the function $y = f(x)$.
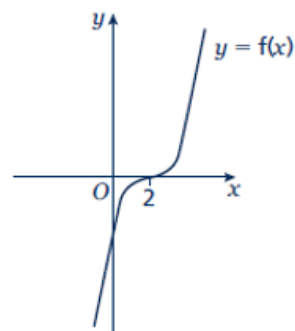
Sketch the graph of $y = f(x - 3)$.

|	
	For the function $y = f(x - 3)$ translate the function $y = f(x)$ 3 units right.

Practice

1 The graph shows the function $y = f(x)$.
Copy the graph and on the same axes sketch and
label the graphs of $y = f(x) + 4$ and $y = f(x + 2)$.

2 The graph shows the function $y = f(x)$.
Copy the graph and on the same axes sketch and
label the graphs of $y = f(x + 3)$ and $y = f(x) - 3$.

3 The graph shows the function $y = f(x)$.
Copy the graph and on the same axes sketch the
graph of $y = f(x - 5)$.
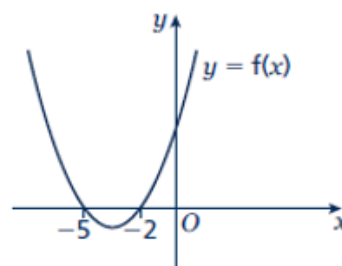
4 The graph shows the function $y = f(x)$ and two transformations of $y = f(x)$, labelled C_1 and C_2. Write down the equations of the translated curves C_1 and C_2 in function form.
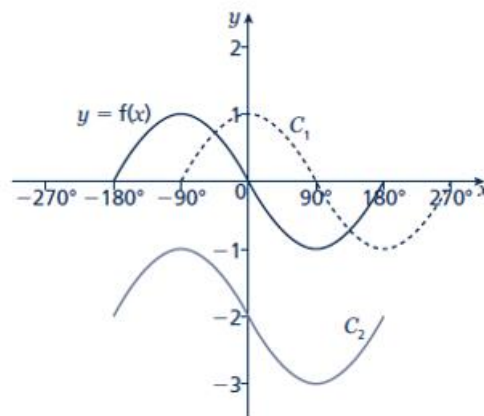
5 The graph shows the function $y = f(x)$ and two transformations of $y = f(x)$, labelled C_1 and C_2. Write down the equations of the translated curves C_1 and C_2 in function form.
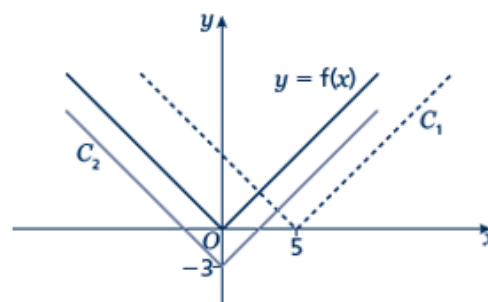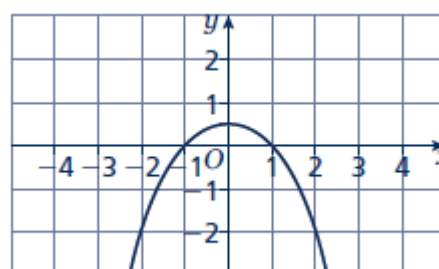

6 The graph shows the function $y = f(x)$.

a Sketch the graph of $y = f(x) + 2$
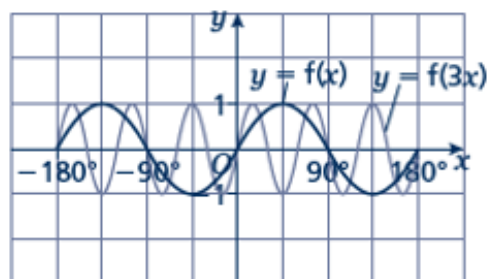**b** Sketch the graph of $y = f(x + 2)$

Stretching graphs

A LEVEL LINKS
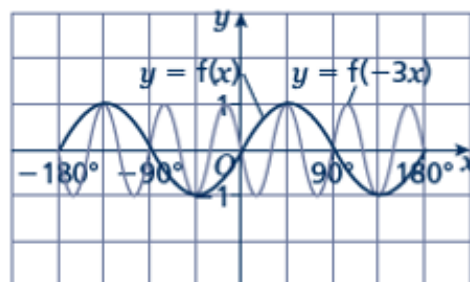**Scheme of work:** 1f. Transformations – transforming graphs – f(*x*) notation
**Textbook:** Pure Year 1, 4.6 Stretching graphs

Key points

- The transformation $y = f(ax)$ is a horizontal stretch of $y = f(x)$ with scale factor $\frac{1}{a}$ parallel to the *x*-axis.
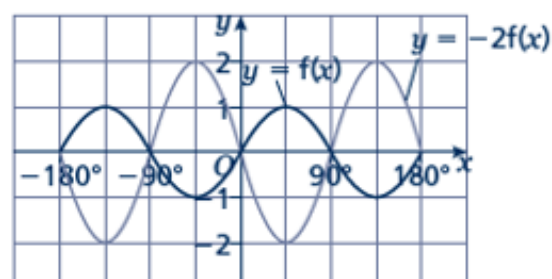

- The transformation $y = f(-ax)$ is a horizontal stretch of $y = f(x)$ with scale factor $\frac{1}{a}$ parallel to the *x*-axis and then a reflection in the *y*-axis.

- The transformation $y = af(x)$ is a vertical stretch of $y = f(x)$ with scale factor *a* parallel to the *y*-axis.
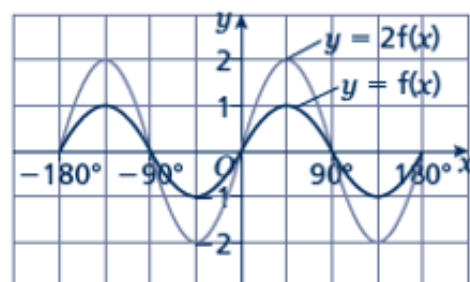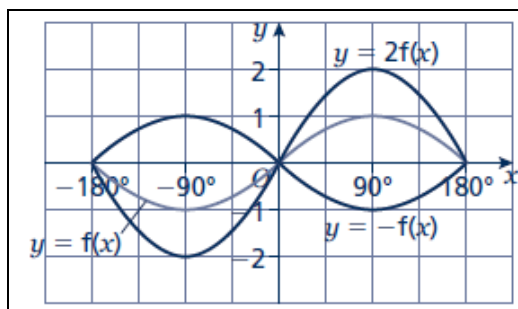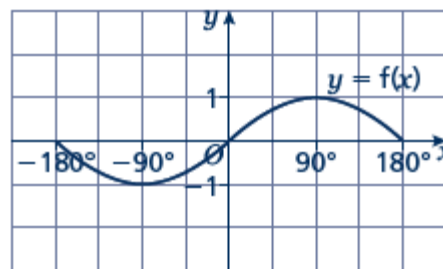

- The transformation $y = -af(x)$ is a vertical stretch of $y = f(x)$ with scale factor *a* parallel to the *y*-axis and then a reflection in the *x*-axis.

Examples

Example 3   The graph shows the function $y = f(x)$.

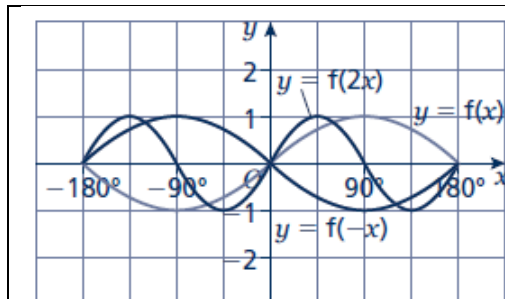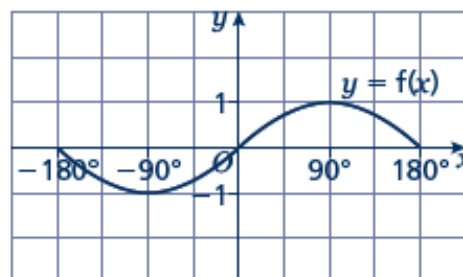Sketch and label the graphs of
$y = 2f(x)$ and $y = -f(x)$.

The function $y = 2f(x)$ is a vertical stretch of $y = f(x)$ with scale factor 2 parallel to the y-axis.

The function $y = -f(x)$ is a reflection of $y = f(x)$ in the x-axis.

Example 4   The graph shows the function $y = f(x)$.

Sketch and label the graphs of
$y = f(2x)$ and $y = f(-x)$.

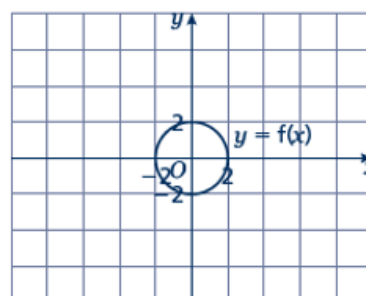
The function $y = f(2x)$ is a horizontal stretch of $y = f(x)$ with scale factor $\frac{1}{2}$ parallel to the x-axis.
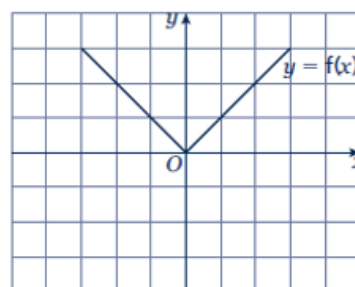
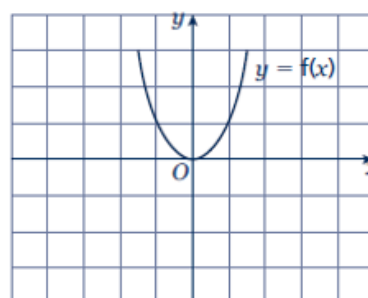The function $y = f(-x)$ is a reflection of $y = f(x)$ in the y-axis.

Practice

7 The graph shows the function $y = f(x)$.

    a Copy the graph and on the same axes sketch and label the graph of $y = 3f(x)$.

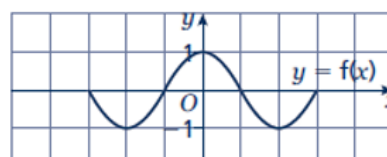   **b** Make another copy of the graph and on the same axes sketch and label the graph of $y = f(2x)$.

8 The graph shows the function $y = f(x)$. Copy the graph and on the same axes sketch and label the graphs of $y = -2f(x)$ and $y = f(3x)$.

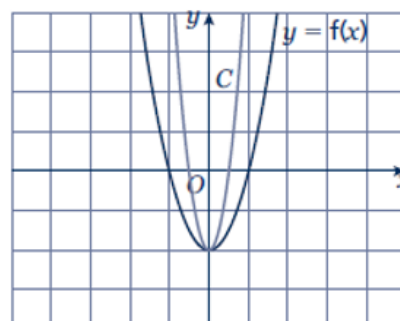**9** The graph shows the function $y = f(x)$. Copy the graph and, on the same axes, sketch and label the graphs of $y = -f(x)$ and $y = f\left(\frac{1}{2}x\right)$.
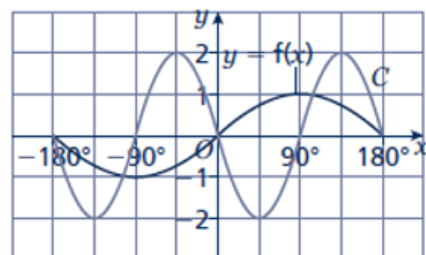

10 The graph shows the function $y = f(x)$. Copy the graph and, on the same axes, sketch the graph of $y = -f(2x)$.
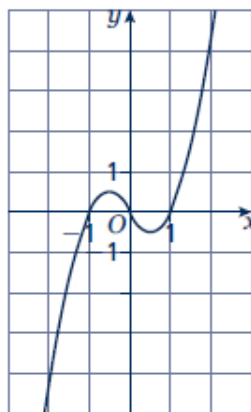

11 The graph shows the function $y = f(x)$ and a transformation, labelled C. Write down the equation of the translated curve C in function form.

12 The graph shows the function $y = f(x)$ and a transformation labelled C.
Write down the equation of the translated curve C in function form.

13 The graph shows the function $y = f(x)$.

   a   Sketch the graph of $y = -f(x)$.

   b   Sketch the graph of $y = 2f(x)$.
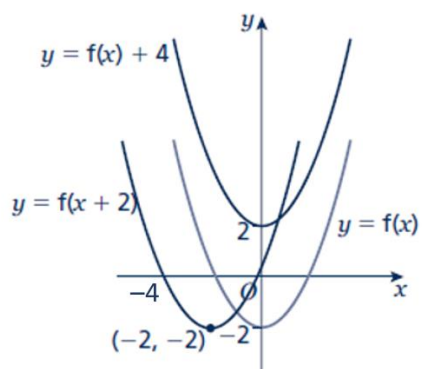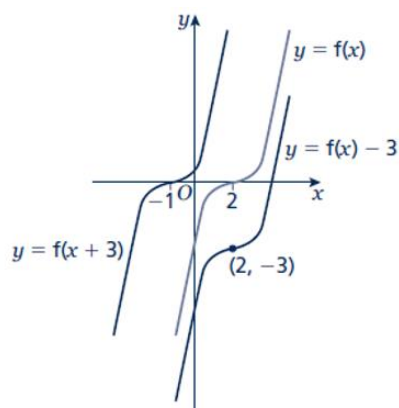

Extend

14 **a**   Sketch and label the graph of $y = f(x)$, where $f(x) = (x - 1)(x + 1)$.

     b   On the same axes, sketch and label the graphs of $y = f(x) - 2$ and $y = f(x + 2)$.

15 **a**   Sketch and label the graph of $y = f(x)$, where $f(x) = -(x + 1)(x - 2)$.

     b   On the same axes, sketch and label the graph of $y = f\left(-\frac{1}{2}x\right)$.
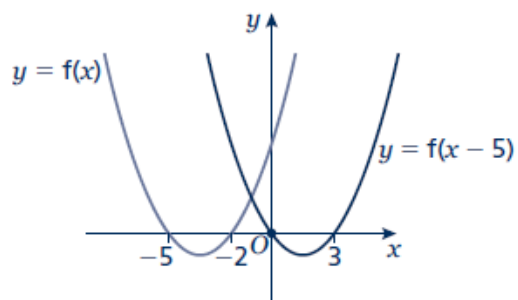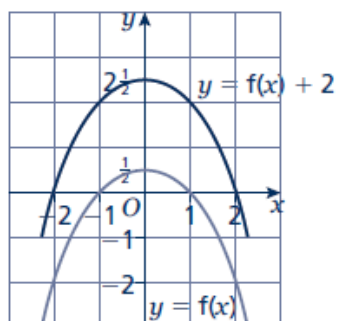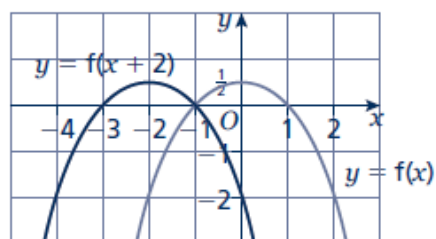
Answers

1

2

3

4 C_1: $y = f(x - 90°)$
C_2: $y = f(x) - 2$

5 C_1: $y = f(x - 5)$
C_2: $y = f(x) - 3$
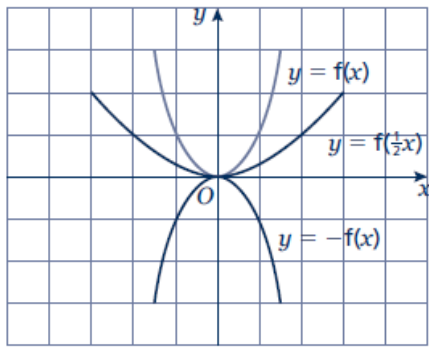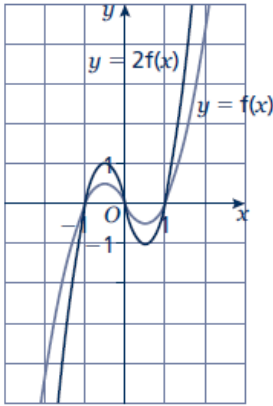
6 **a** **b**

7 **a**
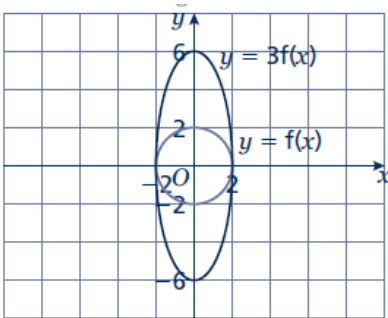
b
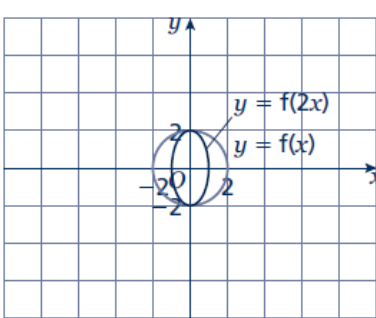
8
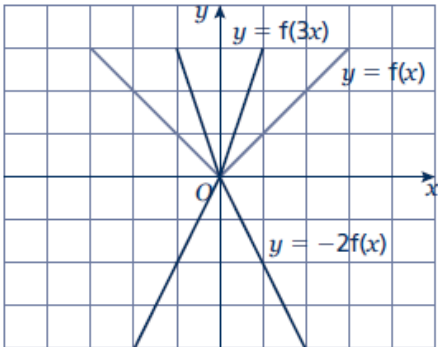
9

10
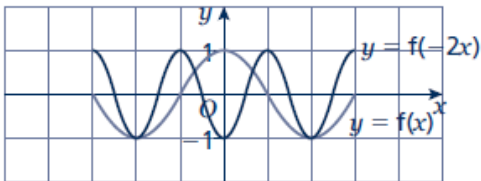
11 $y = f(2x)$

12 $y = -2f(2x)$ or $y = 2f(-2x)$

13 **a**

b

14

15